

Multiple Point Principle of the Standard Model with Scalar Singlet Dark Matter and Right Handed Neutrinos

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Abstract

We consider the multiple point principle (MPP) of the Standard Model (SM) with the scalar singlet Dark Matter (DM) and three heavy right-handed neutrinos at the scale where the beta function β_λ of the effective Higgs self coupling λ_{eff} becomes zero. We make the two-loop analysis and find that the top quark mass M_t and the Higgs portal coupling κ are strongly related each other. One of the good points in this model is that the larger M_t ($\gtrsim 171\text{GeV}$) is allowed. This fact is consistent with the recent experimental value [28] $M_t = 173.34 \pm 0.76 \text{ GeV}$, which corresponds to the DM mass $769 \text{ GeV} \leq m_{\text{DM}} \leq 1053 \text{ GeV}$.

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1 Introduction

The discovery of the Higgs like particle and its mass [1, 2] is a very meaningful result for the Standard Model (SM). It suggests that the Higgs potential can be stable up to the Planck scale M_{pl} and also that both of the Higgs self coupling λ and its beta function β_λ become very small around the Planck scale. This fact attracts much attention, and there are many works which try to find its physical meaning [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

One of the interesting and meaningful studies is to consider how the physics beyond the SM affects such a criticality. For example, recently there has been a two loop analysis about the Higgs portal Z_2 scalar model [23]. In this model, the SM singlet scalar is a Dark Matter (DM) candidate, and it is found that its mass can be predicted to be $400\text{GeV} < m_{DM} < 470\text{GeV}$ from the requirement that λ and β_λ simultaneously become zero at 10^{17}GeV , which is usually called the multiple point principle (MPP) [3, 4, 5, 6].

In this paper, we study the MPP of the next minimal extension of the SM, namely, besides the Higgs portal Z_2 scalar, we include SM singlet heavy right-handed neutrinos [20, 24, 25]. The MPP of this model at the (reduced) Planck scale M_{pl} has already been investigated in [20]. There, by using the two-loop beta functions and the tree-level Higgs potential, they concluded that m_{DM} and the heavy Majorana mass M_R of the right-handed neutrino should be

$$8.5 (8.0) \times 10^2 \text{ GeV} \leq m_{DM} \leq 1.4 (1.2) \times 10^3 \text{ GeV}, \quad (1)$$

$$6.3 (5.5) \times 10^{13} \text{ GeV} \leq M_R \leq 1.6 (1.2) \times 10^{14} \text{ GeV}, \quad (2)$$

within $172.6 \text{ GeV} \leq M_t \leq 174.1 \text{ GeV}$. The different points in this paper are as follows:

1. We consider the MPP at the scale where β_λ becomes zero. Namely, we do not fix the MPP scale at M_{pl} . As a result, the condition $\beta_\lambda = 0$ does not reduce the degrees of freedom of parameters.
2. In addition to the two-loop beta functions, we also calculate the one-loop effective potential.
3. We fix M_R to 10^{13} GeV , and include the Yukawa coupling Y_R between the Z_2 scalar and the right-handed neutrinos.

Although, within the renormalizable Lagrangian, there are also two scalar couplings in this model (see Eq.(12)), we focus on λ (and β_λ) in this paper ¹. The existence of heavy right-handed neutrinos is naturally needed if we try to explain the light neutrino

¹It is difficult to realize the MPP of the other scalar couplings simultaneously in addition to λ . This is discussed in Appendix B.

masses by the seesaw mechanism. Thus, this model is phenomenologically interesting because it can explain both of DM and the light neutrino masses.

This paper is organized as follows. In Section2, we review the MPP of the pure SM for the later discussion. In Section3, we give the two-loop analysis of the SM with the scalar singlet DM and three right handed neutrinos. In Section4, the summary is given.

2 Preliminary - Multiple Point Principle of SM -

In the SM, the one loop effective potential in Landau gauge is given by

$$V_{\text{eff}}(\phi, \mu) = V_{\text{tree}}(\phi, \mu) + V_{\text{1loop}}^{\text{SM}}(\phi, \mu), \quad (3)$$

where

$$V_{\text{tree}}(\phi, \mu) := e^{4\Gamma(\phi)} \frac{\lambda(\mu)}{4} \phi^4, \quad (4)$$

$$V_{\text{1loop}}(\phi) := e^{4\Gamma(\phi)} \left\{ -6 \cdot \frac{M_t(\phi)^4}{64\pi^2} \left[\log \left(\frac{M_t^2(\phi)}{\mu^2} \right) - \frac{3}{2} + 2\Gamma(\phi) \right] \right. \\ \left. + 3 \cdot \frac{M_W(\phi)^4}{64\pi^2} \left[\log \left(\frac{M_W^2(\phi)}{\mu^2} \right) - \frac{5}{6} + 2\Gamma(\phi) \right] + 3 \cdot \frac{M_Z(\phi)^4}{64\pi^2} \left[\log \left(\frac{M_Z^2(\phi)}{\mu^2} \right) - \frac{5}{6} + 2\Gamma(\phi) \right] \right\}, \quad (5)$$

$$M_t(\phi) = \frac{y_t(\mu)}{\sqrt{2}} \phi, \quad M_W(\phi) = \frac{g_2(\mu)}{2} \phi, \quad M_Z = \frac{\sqrt{g_2^2(\mu) + g_Y^2(\mu)}}{2} \phi. \quad (6)$$

Here, μ is the renormalization scale, $\Gamma(\phi)$ is the wave function renormalization and $\lambda(\mu)$, $y_t(\mu)$, $g_2(\mu)$ and $g_Y(\mu)$ are the renormalized couplings². By using those results, the effective Higgs self coupling $\lambda_{\text{eff}}(\phi, \mu)$ can be defined as

$$V_{\text{eff}}(\phi, \mu) := \frac{\lambda_{\text{eff}}(\phi, \mu)}{4} \phi^4. \quad (7)$$

To minimize the contribution of $V_{\text{1loop}}^{\text{SM}}(\phi, \mu)$, we put $\phi = \mu$ in the following discussion.

The left panel of Fig.1 shows $\lambda_{\text{eff}}(\phi)$ as a function of ϕ . For the initial values, we have used the numerical results of [26], and the Higgs mass is fixed at

$$M_h = 125.15 \text{ GeV}. \quad (8)$$

²For the beta functions of the SM, see [23, 26, 31] for example. Or we can reproduce them by using the results in Appendix A.

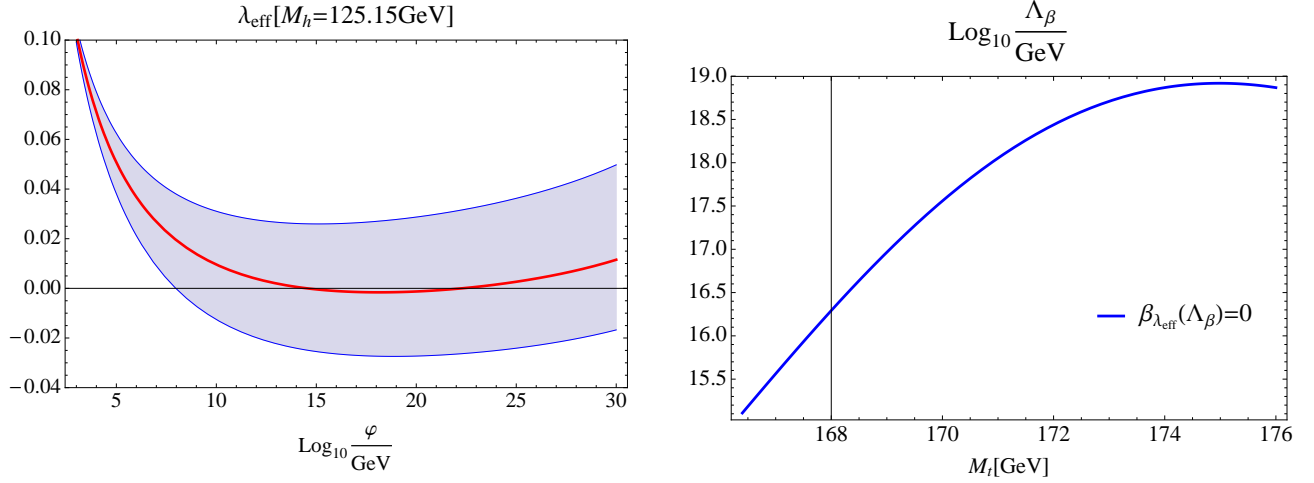


Figure 1: Left panel shows the running effective Higgs self coupling λ_{eff} as a function of the Higgs field ϕ . The blue band corresponds to 95% CL deviation of the top quark pole mass M_t . Right panel shows the scale Λ_β where $\beta_{\lambda_{\text{eff}}}$ becomes zero as a function of M_t .

We use Eq.(8) as a typical value in the following discussion. The band corresponds 95% CL deviation of the top quark pole mass M_t . For the 1σ level, this is given by [27]

$$M_t = 171.2 \pm 2.4 \text{ GeV}. \quad (9)$$

If we assume that all the other parameters of the SM except for M_t are fixed, we can find the scale Λ_β where $\beta_{\lambda_{\text{eff}}}$ becomes zero as a function of M_t . Here, $\beta_{\lambda_{\text{eff}}}$ means

$$\beta_{\lambda_{\text{eff}}}(\phi) := \frac{d\lambda_{\text{eff}}(\phi)}{d\log \phi}. \quad (10)$$

The right panel of Fig.1 shows Λ_β as a function of M_t . The MPP requires that $\lambda_{\text{eff}}(\Lambda_\beta)$ should become zero, and predicts

$$M_t = 170.9 \text{ GeV}. \quad (11)$$

This is the MPP of the pure SM. In the next section, we discuss the MPP of the SM with the scalar singlet DM and three right-handed neutrinos.

3 MPP of the SM with Scalar Singlet Dark Matter and Right Handed Neutrinos

We consider the following renormalizable Lagrangian:

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}\partial_\mu S\partial^\mu S - \frac{m_{\text{DM}}^2}{2}S^2 - \frac{\kappa}{2}S^2 H^\dagger H - \frac{\lambda_{\text{DM}}}{4!}S^4 + \sum_{j=1}^3 \bar{\nu}_{Rj} i\gamma^\mu \partial_\mu \nu_{Rj} \\ - \sum_{i,j} (y_{\nu ij} \bar{L}_i H^\dagger \nu_{Rj} + \text{h.c.}) - \sum_{i,j} \left(M_{Rij} + \frac{Y_{Rij}}{\sqrt{2}} S \right) \bar{\nu}_{Ri}^c \nu_{Rj}. \end{aligned} \quad (12)$$

Here, H is the Higgs field, S is the SM singlet real scalar field, m_{DM} is its mass, ν_{Ri} are right-handed neutrinos, M_{Rij} are their Majorana masses, and $(Y_{Rij}, y_{\nu ij})$ are the Yukawa couplings. For simplicity, we assume that M_{Rij} , Y_{Rij} and $y_{\nu ij}$ are diagonalized, and also that they are equal respectively for the three generations. In this case, Eq.(12) becomes

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}\partial_\mu S\partial^\mu S - \frac{m_{\text{DM}}^2}{2}S^2 - \frac{\kappa}{2}S^2 H^\dagger H - \frac{\lambda_{\text{DM}}}{4!}S^4 + \sum_{i=1}^3 \bar{\nu}_{Ri} i\gamma^\mu \partial_\mu \nu_{Ri} \\ - y_\nu \sum_{i=1}^3 (\bar{L}_i H^\dagger \nu_{Ri} + \text{h.c.}) - \sum_{i=1}^3 \left(M_R + \frac{Y_R}{\sqrt{2}} S \right) \bar{\nu}_{Ri}^c \nu_{Ri}. \end{aligned} \quad (13)$$

Thus, including the top mass M_t , there are seven unknown parameters

$$M_t, \quad m_{\text{DM}}, \quad \kappa, \quad \lambda_{\text{DM}}, \quad y_\nu, \quad M_R, \quad Y_R, \quad (14)$$

in this model. In the following discussion, to distinguish the initial values of these parameters at $\mu = M_t$ from their running couplings, we put the subscript 0 for their initial values, like κ_0 except for M_t . Because S is the candidate of the DM, m_{DM} and κ must satisfy some relation such that they can explain the observed energy density [29]

$$\Omega_{DM} h^2 := \frac{\rho_{\text{DM}} h^2}{\rho_{\text{tot}}} = 0.1196 \pm 0.0031 (68\% \text{ CL}). \quad (15)$$

For $m_{\text{DM}} \gtrsim M_h$, this relation is approximately given by [30]

$$\log_{10} \kappa \simeq -3.63 + 1.04 \log_{10} \left(\frac{m_{\text{DM}}}{\text{GeV}} \right). \quad (16)$$

Moreover, if we assume that the neutrino mass is 0.1eV, y_ν and M_R must satisfy

$$-\frac{M_R}{2} \left(1 - \sqrt{1 + \frac{2y_\nu^2 v_h^2}{M_R^2}} \right) \simeq \frac{y_\nu^2 v_h^2}{2M_R} = 0.1\text{eV}, \quad (17)$$

where v_h is the Higgs expectation value. This is the usual relation of the seesaw mechanism. In the following discussion, we choose $M_R = 10^{13}\text{GeV}$, so y_ν is fixed by Eq.(17). As a result, among the seven parameters, four of them remain as free parameters; they are

$$M_t, \kappa, \lambda_{\text{DM}} \text{ and } Y_R. \quad (18)$$

To discuss how the effective couplings behave at the high energy scale, we must know the renormalization group equations (RGEs) of this model. Their results are presented in Appendix A. Here, note that the contributions from the heavy right-handed neutrinos should be taken into account at the scale where $\mu \geq M_R$. The 1-loop effective potential of the Higgs field is given by

$$V_{\text{1loop}}(\phi, \mu) := \begin{cases} V_{\text{1loop}}^{\text{SM}}(\phi) + \frac{M_{\text{DM}}(\phi)^4}{64\pi^2} \left[\log \left(\frac{M_{\text{DM}}(\phi)^2}{\mu^2} \right) - \frac{3}{2} \right] - 6 \cdot \frac{M_\nu^-(\phi)^4}{64\pi^2} \left[\log \left(\frac{M_\nu^-(\phi)^2}{\mu^2} \right) - \frac{3}{2} \right] & (\text{for } \phi < M_R), \\ V_{\text{1loop}}^{\text{SM}}(\phi) + \frac{M_{\text{DM}}(\phi)^4}{64\pi^2} \left[\log \left(\frac{M_{\text{DM}}(\phi)^2}{\mu^2} \right) - \frac{3}{2} \right] - 6 \cdot \frac{M_\nu^-(\phi)^4}{64\pi^2} \left[\log \left(\frac{M_\nu^-(\phi)^2}{\mu^2} \right) - \frac{3}{2} \right] \\ - 6 \cdot \frac{M_\nu^+(\phi)^4}{64\pi^2} \left[\log \left(\frac{M_\nu^+(\phi)^2}{\mu^2} \right) - \frac{3}{2} \right] & (\text{for } \phi > M_R), \end{cases} \quad (19)$$

where

$$M_{\text{DM}}(\phi) := \sqrt{e^{2\Gamma(\phi)} \frac{\kappa \phi^2}{2} + m_{\text{DM}}^2}, \quad M_\nu^\pm(\phi) := \frac{M_R}{2} \left(1 \pm \sqrt{1 + \frac{2y_\nu^2 e^{2\Gamma(\phi)} \phi^2}{M_R^2}} \right). \quad (20)$$

In these expressions, we have put $S = 0$ because we now focus on the MPP of the Higgs sector³. Furthermore, we can neglect m_{DM} in Eq.(20) because its effect is very small when $\phi \gg m_{\text{DM}}$. As well as Section 2, we put $\phi = \mu$, and define the effective Higgs self coupling λ_{eff} as

$$\lambda_{\text{eff}}(\phi) := \frac{4}{\phi^4} V(\phi) = \frac{4}{\phi^4} (V_{\text{tree}}(\phi) + V_{\text{1loop}}(\phi)). \quad (21)$$

Fig.2 shows $\lambda_{\text{eff}}(\phi)$ for the various values of parameters. Here, the typical values are chosen to be

$$\lambda_{\text{DM}0} = 0.2, \quad \kappa_0 = 0.2, \quad Y_{R0} = 0.2. \quad (22)$$

One can see that λ_{eff} depends mainly on M_t and κ_0 , and hardly on $\lambda_{\text{DM}0}$ and Y_{R0} . This is because λ_{DM} does not appear in β_λ and Y_R appears at the two-loop level (see Eq.(33) in Appendix A). Therefore, by fixing λ_{DM} and Y_R , we can relate M_t and κ_0 from the MPP.

³Of course, we can consider the MPP of the DM sector. We study such situation in Appendix B.

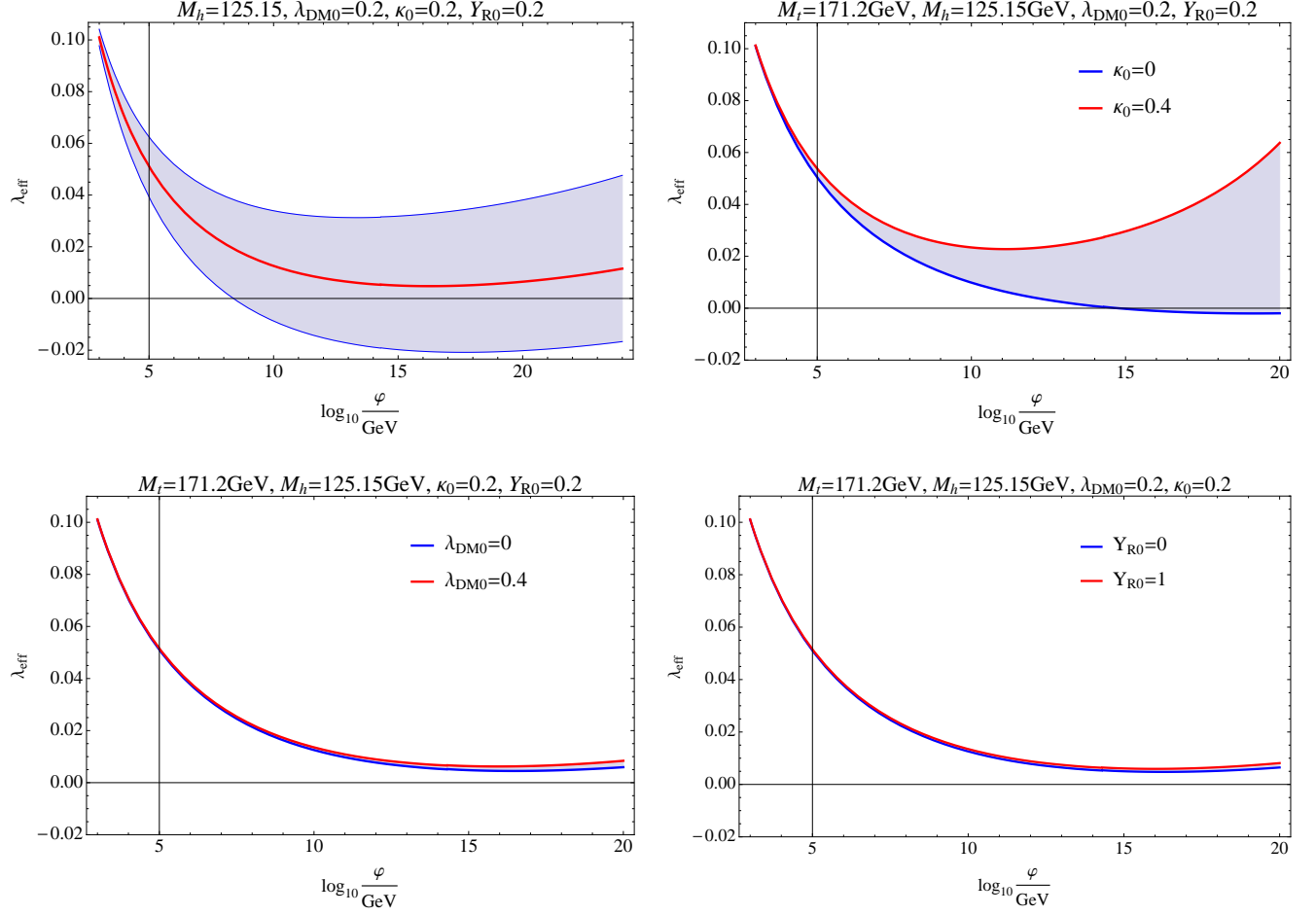


Figure 2: The running effective Higgs self coupling λ_{eff} as a function of ϕ . The upper left (right) panel shows the M_t (κ_0) dependence. For M_t , the blue band corresponds 95% CL deviation from 171.2 GeV. The lower left (right) panel shows the λ_{DM0} (Y_{R0}) dependence.

By the same procedure of Section 2, we can calculate the scale Λ_β where $\beta_{\lambda_{\text{eff}}}$ becomes zero, and obtain $\lambda_{\text{eff}}(\Lambda_\beta)$ as a function of M_t and κ_0 . Fig. 3 shows the results. In the upper (lower) panels, Y_{R0} is fixed to 0.2 (0.7). The difference between the left and right panels is whether the tree or one-loop level potential is used. The parameter region where $\lambda_{\text{eff}}(\Lambda_\beta) < 0$ and $\lambda_{\text{DM}}(\Lambda_\beta) < 0$ are filled respectively by blue and red. Both of them are excluded from the stability of the potentials. The MPP predicts that M_t and κ_0 should exist on the green contour. One of the good points of this model is that the larger M_t is allowed unlike the SM. This is consistent with the recent experimental value [28]

$$M_t = 173.34 \pm 0.76 \text{ GeV}, \quad (23)$$

which corresponds to the DM mass (see Eq. (16))

$$769 \text{ GeV} \leq m_{\text{DM}} \leq 1053 \text{ GeV}. \quad (24)$$

Two comments are needed.

1. The contours which represent $\Lambda_\beta = 10^{16} \text{ GeV}$, 10^{17} GeV and 10^{18} GeV are also shown in Fig. 3 respectively by red, blue and orange. Thus, the larger M_t (such as Eq. (23)) means that, in this model, the MPP of the Higgs potential occurs at the relatively low energy scale ($\lesssim 10^{16} \text{ GeV}$).
2. As is seen from the lower panels of Fig. 3, we can also require $\lambda_{\text{DM}}(\Lambda_\beta) = 0$ in addition to $\lambda_{\text{eff}}(\Lambda_\beta) = 0$. Because κ_0 and Y_{R0} appear in the one-loop part of $\beta_{\lambda_{\text{DM}}}$, we can obtain a further relation between them by $\lambda_{\text{DM}}(\Lambda_\beta) = 0$. Although one might think that the remaining one parameter can be determined by $\beta_{\lambda_{\text{DM}}}(\Lambda_\beta) = 0$, we have checked that it is difficult to satisfy $\lambda_{\text{DM}}(\Lambda_\beta) = \beta_{\lambda_{\text{DM}}}(\Lambda_\beta) = 0$ simultaneously. See Appendix B for more details.

4 Summary

We have discussed the MPP of the SM with the scalar singlet DM and right-handed neutrinos. We have found that λ_{eff} and $\beta_{\lambda_{\text{eff}}}$ can simultaneously become zero within the reasonable parameter region. The MPP predicts the strong relation between the portal coupling κ and the top mass M_t . Unlike the pure SM, the larger M_t is allowed in this model, which is favorable for the recent experimental values [27, 28]

$$M_t = 173.34 \pm 0.76 \text{ GeV}. \quad (25)$$

Although we have found that the MPP can be satisfied for the Higgs potential, it is difficult to realize the exact flatness of the scalar potential at some high energy scale Λ ;

$$\lambda(\Lambda) = \beta_\lambda(\Lambda) = \lambda_{\text{DM}}(\Lambda) = \beta_{\lambda_{\text{DM}}}(\Lambda) = \kappa(\Lambda) = \beta_\kappa(\Lambda) = 0. \quad (26)$$

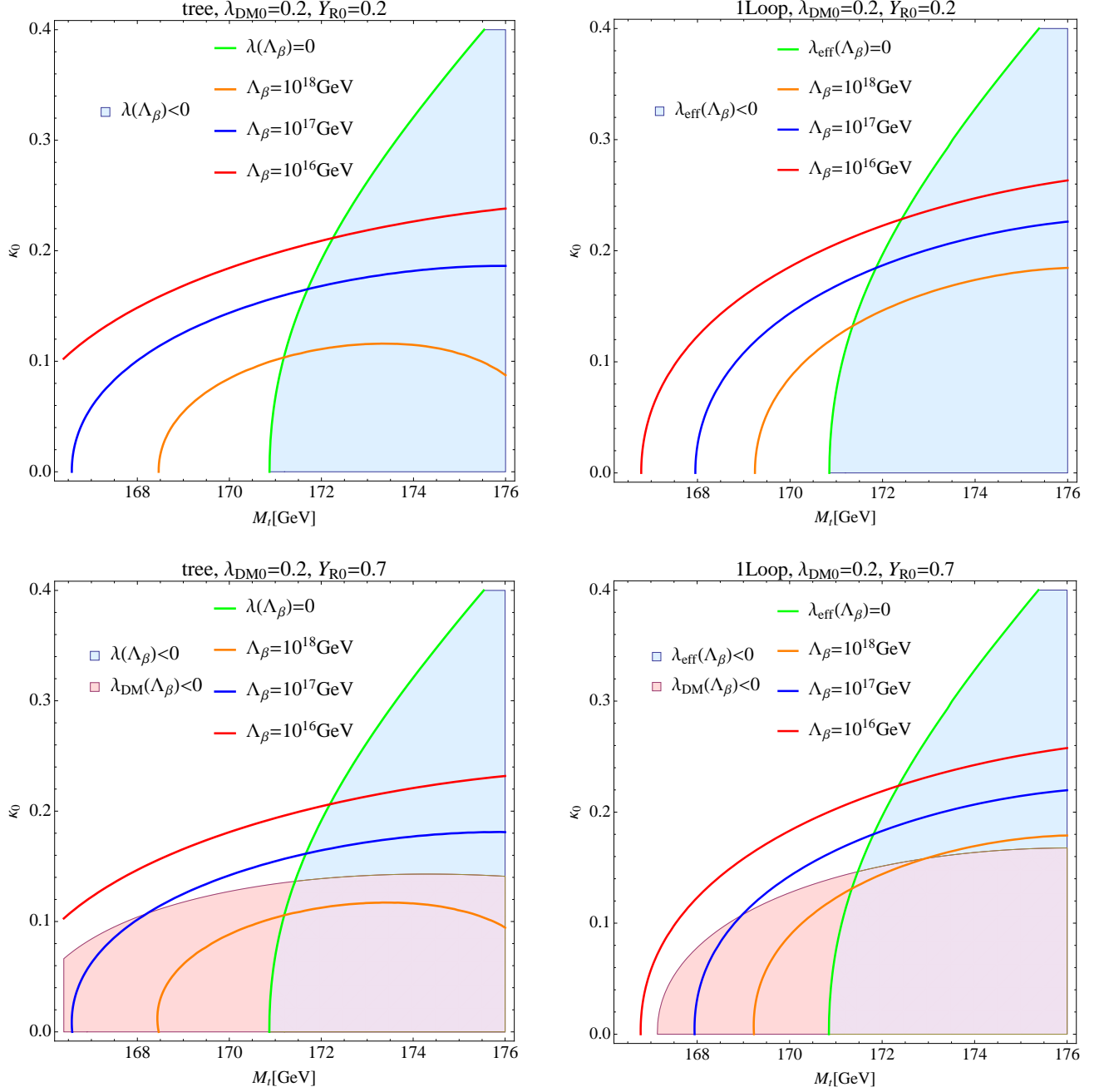


Figure 3: The parameter dependences of $\lambda_{\text{eff}}(\Lambda_\beta)$. Here, $\lambda_{\text{DM}0}$ is fixed to 0.2, and Y_{R0} is fixed to 0.2(0.7) in the upper (lower) panels. The left (right) panels show the calculations by using the tree (one-loop) level potential. The green lines are the prediction by the MPP. The contours which represent $\Lambda_\beta = 10^{16}\text{GeV}$, 10^{17}GeV and 10^{18}GeV are also shown respectively by red, blue and orange.

See Appendix B for the details. It would be interesting to consider a generalization of this model in such a way that the MPP can be realized for the whole scalar fields.

Acknowledgement

We thank Hikaru Kawai, Yuta Hamada and Koji Tsumura for valuable discussions.

Appendix A Two Loop Renormalization Group Equations

The two loop RGEs where the Lagrangian is given by Eq.(13) are as follows:⁴

$$\frac{dg_Y}{dt} = \frac{1}{(4\pi)^2} \frac{41}{6} g_Y^3 + \frac{g_Y^3}{(4\pi)^4} \left(\frac{199}{18} g_Y^2 + \frac{9}{2} g_2^2 + \frac{44}{3} g_3^2 - \frac{17}{6} y_t^2 - \frac{3}{2} y_\nu^2 \right), \quad (27)$$

$$\frac{dg_2}{dt} = -\frac{1}{(4\pi)^2} \frac{19}{6} g_2^3 + \frac{g_2^3}{(4\pi)^4} \left(\frac{3}{2} g_Y^2 + \frac{35}{6} g_2^2 + 12 g_3^2 - \frac{3}{2} (y_t^2 + y_\nu^2) \right), \quad (28)$$

$$\frac{dg_3}{dt} = -\frac{7}{(4\pi)^2} g_3^3 + \frac{g_3^3}{(4\pi)^4} \left(\frac{11}{6} g_Y^2 + \frac{9}{2} g_2^2 - 26 g_3^2 - 2 y_t^2 \right), \quad (29)$$

$$\begin{aligned} \frac{dy_t}{dt} = & \frac{y_t}{(4\pi)^2} \left(\frac{9}{2} y_t^2 + 3 y_\nu^2 - \frac{17}{12} g_Y^2 - \frac{9}{4} g_2^2 - 8 g_3^2 \right) \\ & + \frac{y_t}{(4\pi)^4} \left\{ -12 y_t^4 - \frac{27}{4} y_\nu^4 - \frac{27}{4} y_t^2 y_\nu^2 - \frac{9}{8} Y_R^2 y_\nu^2 + 6 \lambda^2 + \frac{1}{4} \kappa^2 - 12 \lambda y_t^2 + g_Y^2 \left(\frac{131}{16} y_t^2 + \frac{15}{8} y_\nu^2 \right) \right. \\ & \left. + g_2^2 \left(\frac{225}{16} y_t^2 + \frac{45}{8} y_\nu^2 \right) + 36 g_3^2 y_t^2 + \frac{1187}{216} g_Y^4 - \frac{23}{4} g_2^4 - 108 g_3^4 - \frac{3}{4} g_Y^2 g_2^2 + 9 g_2^2 g_3^2 + \frac{19}{9} g_3^2 g_Y^2 \right\}, \end{aligned} \quad (30)$$

$$\begin{aligned} \frac{dy_\nu}{dt} = & \frac{y_\nu}{(4\pi)^2} \left(\frac{9}{2} y_\nu^2 + 3 y_t^2 + \frac{1}{4} Y_R^2 - \frac{3}{4} g_Y^2 - \frac{9}{4} g_2^2 \right) \\ & + \frac{y_\nu}{(4\pi)^4} \left\{ -12 y_\nu^4 - \frac{27}{4} y_t^4 - \frac{19}{32} Y_R^4 - y_\nu^2 \left(\frac{27}{4} y_t^2 + \frac{21}{16} Y_R^2 \right) + 6 \lambda^2 + \frac{1}{4} \kappa^2 - 12 \lambda y_\nu^2 - \kappa Y_R^2 \right. \\ & + g_Y^2 \left(\frac{123}{16} y_\nu^2 + \frac{85}{24} y_t^2 + \frac{9}{16} Y_R^2 \right) + g_2^2 \left(\frac{225}{16} y_\nu^2 + \frac{45}{8} y_t^2 + \frac{27}{16} Y_R^2 \right) + 20 g_3^2 y_t^2 \\ & \left. + \frac{35}{24} g_Y^4 - \frac{23}{4} g_2^4 - \frac{9}{4} g_Y^2 g_2^2 \right\}, \end{aligned} \quad (31)$$

$$\begin{aligned} \frac{dY_R}{dt} = & \frac{Y_R}{(4\pi)^2} (3 Y_R^2 + 2 y_\nu^2) + \frac{Y_R}{(4\pi)^4} \left\{ -\frac{81}{16} Y_R^4 - \frac{27}{4} Y_R^2 y_\nu^2 - 9 y_t^2 y_\nu^2 - \frac{27}{2} y_\nu^4 + \frac{1}{12} \lambda_{\text{DM}}^2 + \kappa^2 \right. \\ & \left. - \lambda_{\text{DM}} Y_R^2 - 8 \kappa y_\nu^2 - \frac{1}{4} g_Y^2 y_\nu^2 - \frac{3}{4} g_2^2 y_\nu^2 \right\}, \end{aligned} \quad (32)$$

⁴The calculations in this appendix are based on [32, 33, 34], and our results are in agreement with the recent result [25] when there is only one right-handed neutrino and $Y_R = 0$.

$$\begin{aligned}
\frac{d\lambda}{dt} = & \frac{1}{(4\pi)^2} \left(\lambda (24\lambda - 9g_2^2 - 3g_Y^2 + 12y_\nu^2 + 12y_t^2) + \frac{3}{4}g_2^2g_Y^2 + \frac{9g_2^4}{8} + \frac{3g_Y^4}{8} + \frac{\kappa^2}{2} - 6y_\nu^4 - 6y_t^4 \right) \\
& + \frac{1}{(4\pi)^4} \left\{ -2\kappa^3 - 5\kappa^2\lambda - 312\lambda^3 + 36\lambda^2 (g_Y^2 + 3g_2^2) + \lambda \left(\frac{629}{24}g_Y^4 + \frac{39}{4}g_2^2g_Y^2 - \frac{73}{8}g_2^4 \right) \right. \\
& + \frac{305}{16}g_2^6 - \frac{289}{48}g_Y^2g_2^4 - \frac{559}{48}g_Y^4g_2^2 + \frac{379}{48}g_Y^6 - 32g_3^2y_t^4 - \frac{8}{3}g_Y^2y_t^4 - \frac{9}{4}g_2^4(y_t^2 + y_\nu^2) \\
& + \lambda y_t^2 \left(\frac{85}{6}g_Y^2 + \frac{45}{2}g_2^2 + 80g_3^2 \right) + \lambda y_\nu^2 \left(\frac{15}{2}g_Y^2 + \frac{45}{2}g_2^2 \right) + g_Y^2y_t^2 \left(-\frac{19}{4}g_Y^2 + \frac{21}{2}g_2^2 \right) \\
& - g_Y^2y_\nu^2 \left(\frac{3}{4}g_Y^2 + \frac{3}{2}g_2^2 \right) - 144\lambda^2 (y_t^2 + y_\nu^2) - 3\lambda \left(y_t^4 + y_\nu^4 + \frac{3}{2}Y_R^2y_\nu^2 \right) \\
& \left. + 30 \left(y_t^6 + y_\nu^6 + \frac{1}{5}Y_R^2y_\nu^4 \right) - \frac{3}{2}Y_R^2\kappa^2 \right\}, \tag{33}
\end{aligned}$$

$$\begin{aligned}
\frac{d\lambda_{\text{DM}}}{dt} = & \frac{1}{(4\pi)^2} (3\lambda_{\text{DM}}^2 + 12\kappa^2 + 6\lambda_{\text{DM}}Y_R^2 - 18Y_R^4) \\
& + \frac{1}{(4\pi)^4} \left\{ -\frac{17}{3}\lambda_{\text{DM}}^3 - 20\kappa^2\lambda_{\text{DM}} - 48\kappa^3 - 72(y_t^2 + y_\nu^2)\kappa^2 + 24(g_Y^2 + 3g_2^2)\kappa^2 \right. \\
& \left. + 72Y_R^4(Y_R^2 + y_\nu^2) + \lambda_{\text{DM}}Y_R^2 \left(\frac{21}{2}Y_R^2 - 18y_\nu^2 \right) - 9Y_R^2\lambda_{\text{DM}}^2 \right\}, \tag{34}
\end{aligned}$$

$$\begin{aligned}
\frac{d\kappa}{dt} = & \frac{1}{(4\pi)^2} \left(4\kappa^2 + 12\kappa\lambda + \kappa\lambda_{\text{DM}} + 3\kappa(2y_t^2 + 2y_\nu^2 + Y_R^2) - \frac{3}{2}\kappa(g_Y^2 + 3g_2^2) - 12Y_R^2y_\nu^2 \right) \\
& + \frac{\kappa}{(4\pi)^4} \left\{ -\frac{21}{2}\kappa^2 - 72\kappa\lambda - 60\lambda^2 - 6\kappa\lambda_{\text{DM}} - \frac{5}{6}\lambda_{\text{DM}}^2 - (y_t^2 + y_\nu^2)(12\kappa + 72\lambda) - 3Y_R^2(2\kappa + \lambda_{\text{DM}}) \right. \\
& - \frac{27}{2}y_t^4 - \frac{27}{2}y_\nu^4 - \frac{3}{4}Y_R^4 + \frac{51}{4}Y_R^2y_\nu^2 + g_Y^2(\kappa + 24\lambda) + 3g_2^2(\kappa + 24\lambda) + y_t^2 \left(\frac{85}{12}g_Y^2 + \frac{45}{4}g_2^2 + 40g_3^2 \right) \\
& + y_\nu^2 \left(\frac{15}{4}g_Y^2 + \frac{45}{4}g_2^2 \right) + \frac{557}{48}g_Y^4 - \frac{145}{16}g_2^4 + \frac{15}{8}g_Y^2g_2^2 \left. \right\} + \frac{Y_R^2y_\nu^2}{(4\pi)^4} \left\{ \frac{3}{2}(g_Y^2 + 3g_2^2) + 27Y_R^2 + 66y_\nu^2 \right\}, \tag{35}
\end{aligned}$$

$$\frac{d\Gamma}{dt} = \frac{1}{(4\pi)^2} \left(\frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 - 3y_t^2 - 3y_\nu^2 \right). \tag{36}$$

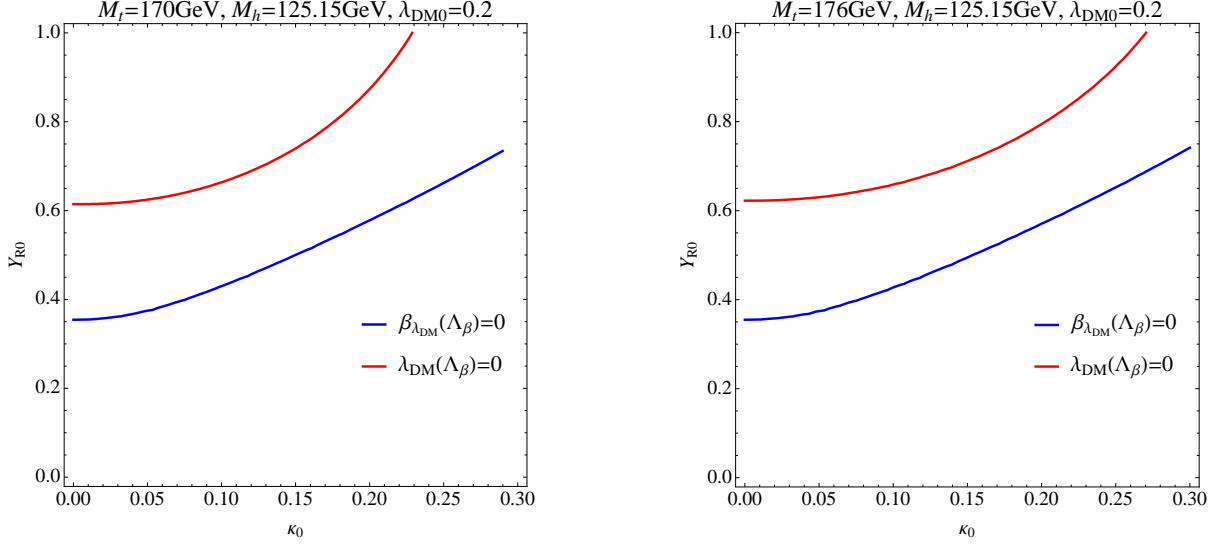


Figure 4: The blue (red) lines show the contours where $\beta_{\lambda_{\text{DM}}}(\lambda_{\text{DM}})(\Lambda_\beta) = 0$. The left (right) panel is the $M_t = 170(176)\text{GeV}$ case. Here, note that if $\kappa_0 \gtrsim 0.3$, Λ_β becomes less than $M_R = 10^{13}\text{ GeV}$, and there is no solution of $\beta_{\lambda_{\text{DM}}}(\Lambda_\beta) = 0$ because the one-loop part of $\beta_{\lambda_{\text{DM}}}$ is always positive when $\mu \leq M_R$.

Appendix B Is Exact Flat Potential Possible?

One of the question is whether the MPP can be realized exactly. Namely,

$$\lambda(\Lambda_\beta) = \beta_\lambda(\Lambda_\beta) = \lambda_{\text{DM}}(\Lambda_\beta) = \beta_{\lambda_{\text{DM}}}(\Lambda_\beta) = \kappa(\Lambda_\beta) = \beta_\kappa(\Lambda_\beta) = 0 \quad (37)$$

is possible or not. Here, for simplicity, we also define Λ_β as the scale where β_λ becomes zero. To discuss this possibility, it is qualitatively enough to consider the one-loop RGEs. One can easily understand it is impossible to realize Eq.(37) as follows; even if $\lambda(\Lambda_\beta)$, $\beta_\lambda(\Lambda_\beta)$, $\lambda_{\text{DM}}(\Lambda_\beta)$ and $\beta_{\lambda_{\text{DM}}}(\Lambda_\beta)$ become simultaneously zero, we can not make $\kappa(\Lambda_\beta)$ zero because the one-loop part of $\beta_{\lambda_{\text{DM}}}$ at Λ_β becomes

$$\beta_{\lambda_{\text{DM}}}|_{\Lambda_\beta} = \frac{1}{(4\pi)^2} (12\kappa^2 - 18Y_R^4), \quad (38)$$

and we need $\kappa(\Lambda_\beta) \neq 0$ to satisfy $\beta_{\lambda_{\text{DM}}}(\Lambda_\beta) = 0$ ⁵. Furthermore, it is also difficult even to satisfy $\lambda_{\text{DM}}(\Lambda_\beta) = \beta_{\lambda_{\text{DM}}}(\Lambda_\beta) = 0$ simultaneously. See Fig.4. This shows the contours such that $\lambda_{\text{DM}}(\Lambda_\beta)$ and $\beta_{\lambda_{\text{DM}}}(\Lambda_\beta)$ become zero respectively. Here, we have used the two-loop RGEs. One can see that two contours do not intersect.

⁵Typically, the non-zero positive Y_{R0} is needed to make $\beta_{\lambda_{\text{DM}}}$ negative. As a result, $Y_R(\Lambda_\beta)$ is non-zero because the one-loop part of β_{Y_R} is always positive.

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